$$= \int_{x=0}^{\pi}\left[\int_{y=0}^{\pi/2} 2e^{-x}\cos y\,dy\right]dx$$

$$\iint_{0\ 0}^{\pi\ \pi/2} (-e^{-x}\cos y - e^{-x}\cos y)\,dx\,dy$$

$$\iint (-2e^{-x}\cos y)\,dx\,dy$$

$$= -2\int_0^{\pi}\left[\int_0^{\pi/2} e^{-x}\cos y\,dy\right]dx$$

$$= -2\int_0^{\pi}\left[e^{-x}\sin y\right]_0^{\pi/2}dx$$

$$= -2\int_0^{\pi} e^{-x}\left[\sin\tfrac{\pi}{2} - \sin 0\right]dx$$

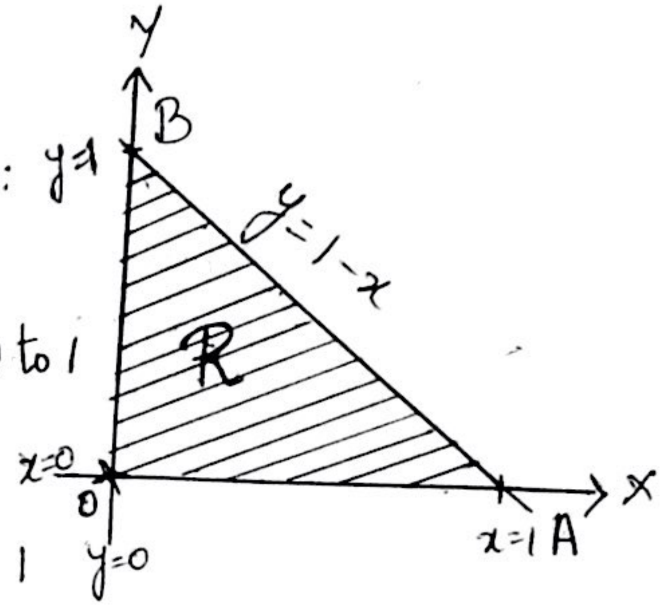
$$= -2\int_0^{\pi} e^{-x}\,dx$$

$$= -2\left[-e^{-x}\right]_0^{\pi} = 2\left[e^{-\pi} - e^{-0}\right]$$

$$= 2\left[e^{-\pi} - 1\right]$$

3) Verify Green's theorem for $\int_C (3x^2-8y^2)\,dx + (4y-6xy)\,dy$

where C is the boundary of the region enclosed by the lines $x=0$, $y=0$, $x+y=1$



Solution: The boundary curve C is made up of three parts:

i) The line OA on which $y=0$, x increases from 0 to 1

ii) The line AB on which $y=1-x$ and x varies from 1 to 0

iii) The line BO on which $x=0$ and y varies from 1 to 0

From given line integral,

$P = 3x^2 - 8y^2$ $\qquad$ $Q = 4y - 6xy$

$\frac{\partial P}{\partial y} = -16y$ $\qquad$ $\frac{\partial Q}{\partial x} = -6y$

By Green's theorem we have,

$$\oint_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx\,dy$$

Consider LHS:

$$\oint_C P\,dx + Q\,dy = \int_{OA} P\,dx + Q\,dy + \int_{AB} P\,dx + Q\,dy + \int_{BO} P\,dx + Q\,dy$$

$$= \int_0^1 3x^2\,dx + \int_1^0 \left[3x^2 - 8(1-x)^2\right] dx + \left[\left[4(1-x) - 6x(1-x)\right](-dx)\right] + \int_1^0 4y\,dy$$

$\left[\because OA \Rightarrow y=0, \quad AB - y = 1-x, \; dy = -dx, \quad BO, \; x = 0, \; dx = 0 \right]$

$$= \int_0^1 3x^2\,dx + \int_1^0 (-11x^2 + 26x - 12)\,dx + \int_1^0 4y\,dy$$

(14)

$$= \left[\cancel{3} \frac{x^3}{\cancel{3}} \right]_0^1 + \left[-11\frac{x^3}{3} + \cancel{26}^{13}\frac{x^2}{\cancel{2}_1} - 12x \right]_1^0 + \left[4\frac{y^2}{2} \right]_1^0$$

$$= \left[x^3 \right]_0^1 + \left[-\frac{11x^3}{3} + 13x^2 - 12x \right]_1^0 + \left[2y^2 \right]_1^0$$

$$= \left[1^3 - 0 \right] + \left[-\frac{11(0)}{3} + 13(0) - 12(0) - \left(-11\frac{(1)^3}{3} + 13(1)^2 - 12(1) \right) \right] + \left[2(0) - 2(1) \right]$$

$$= (1) - \left(-\frac{11}{3} + 13 - 12 \right) + [-2]$$

$$= 1 - \left(-\frac{11}{3} + 1 \right) - 2$$

$$= 1 - \left(-8/3 \right) - 2 = 5/3 \longrightarrow (1)$$

<u>RHS</u>: $$\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx\,dy = \int_{x=0}^{1} \int_{y=0}^{1-x} \left[-6y - (-16y) \right] dy\,dx$$

$$= \int_{x=0}^{1} \int_{y=0}^{1-x} \left[+10y \right] dy\,dx$$

$$= \int_0^1 \left[10 \frac{y^2}{2} \right]_0^{1-x} dx$$

$$= \int_0^1 \left[5y^2 \right]_0^{1-x} dx$$

$$= \int_0^1 \left[5(1-x)^2 - 0 \right] dx$$

$$= \int_0^1 \left[5(1-x)^2 \right] dx$$

$$= \left[-5 \frac{(1-x)^3}{3} \right]_0^1$$

$$= \left[-5 \frac{(1-1)^3}{3} \to 0 - \left(-5 \frac{(1-0)^3}{3} \right) \right]$$

$$= \left[- \left(-5 \left(\tfrac{1}{3} \right) \right) \right]$$

$$= 5/3 \longrightarrow (2)$$

By (1) & (2)

LHS = RHS

Hence Green's theorem is verified.